

# Stochastic Resonance and Anti-cyclonic Rings in the Gulf of Mexico

Benjamín Martínez-López, Jorge Zavala-Hidalgo, and Carlos Gay García

Center for Atmospheric Sciences, National Autonomous University of Mexico,  
Ciudad Universitaria, D.F., Mexico

{benmar, jzavala}@atmosfera.unam.mx, cgay@servidor.unam.mx

**Abstract.** In this work, we used a nonlinear, reduced gravity model of the Gulf of Mexico to study the effect of a seasonal variation of the reduced gravity parameter on ring-shedding behaviour. When small amplitudes of the seasonal variation are used, the distributions of ring-shedding periods are bi-modal. When the amplitude of the seasonal variation is large enough, the ring-shedding events shift to a regime with a constant, yearly period. If the seasonal amplitude of the reduced gravity parameter is small but a noise term is included, then a yearly regime is obtained, suggesting that stochastic resonance could play a role in the ring-shedding process taking place in the Gulf of Mexico.

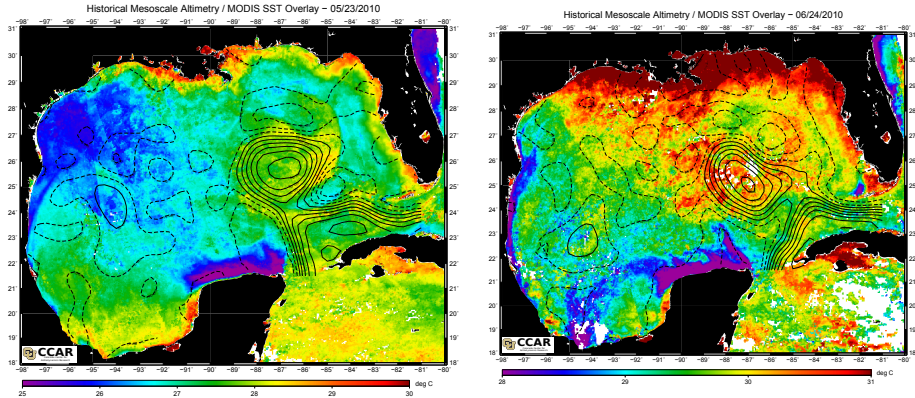
**Keywords:** Gulf of Mexico, Ring Shedding, Reduced Gravity Model, Seasonal Forcing, Stochastic Resonance.

## 1 Introduction

Anti-cyclonic rings generated by meandering of intense boundary current systems are long-lived, intense near-surface features that dominate the oceanic mesoscale in different regions of the World Ocean. They substantially contribute to determine the water mass characteristics as well as the upper-ocean circulation patterns in these regions and, due to their characteristic self-induced, westward propagation, they often also play an important role in the transfer of chemical and biological properties across frontal zones [8].

The circulation in the Gulf of Mexico (GoM) presents two semipermanent traits: the Loop Current (LC) in the eastern region and a cell of anti-cyclonic circulation on the western boundary [6]. The GoM's LC may profoundly influence the local circulation as it moves northward forming meanders, upwelling regions and eventually detached rings (Fig. 1, left panel), which are large, warm, clockwise-spinning vortex of water that migrate westward across the GoM (Fig. 1, right panel) at speeds that have been estimated to range from 2 to 5 km/day [1].

The large surface temperature anomalies as well as the large surface horizontal velocity shears associated with these rings may profoundly influence human activities. For instance, the passage of warm-core rings detached from the LC is able to disturb oil extraction activities, while it is demonstrated that hurricanes may be intensified by their interaction with the warm ring water (see [3] and references therein).

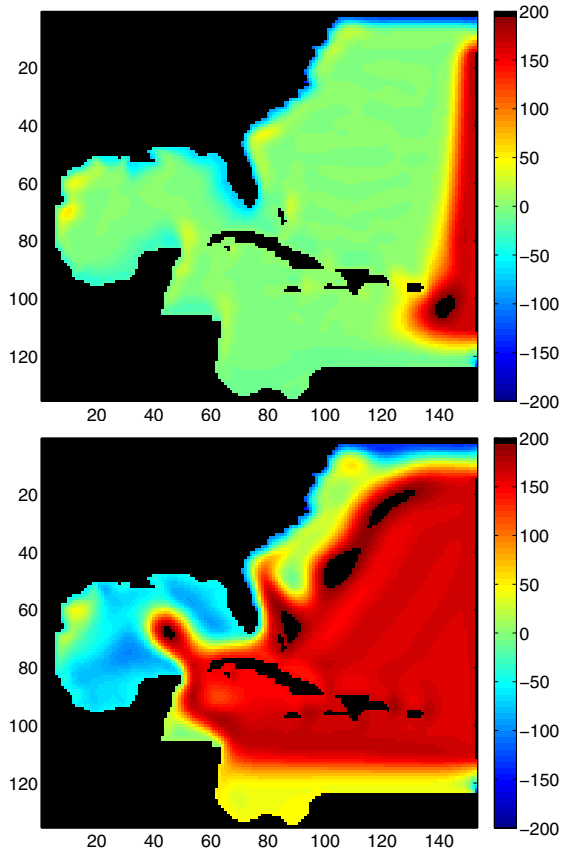


**Fig. 1.** Satellite measurements of sea surface height overlaid with sea surface temperature MODIS data show a detached ring of the Loop Current which starts to move toward the western Gulf of Mexico. Source: [http://eddy.colorado.edu/ccar/ssh/hist\\_gom\\_grid\\_viewer](http://eddy.colorado.edu/ccar/ssh/hist_gom_grid_viewer)

Predicting the onset and evolution of ring shedding in the GoM may substantially contribute to the understanding of the subtle dynamics involved in the local oceanic phenomena and also to the reduction of the impact on human activities caused, directly or indirectly, by these rings. Observations show a nearly bi-modal distribution (the most evident peaks existing around 6 months and 9 to 11 months). Current full-fledged ocean numerical models can explain some of the observed ring-shedding variability but fail in simulating observed periods [5, 11, 9].

Simple models, on the other hand are only able to reproduce an almost constant period [4, 10, 7], which was called the natural period of the Gulf by Hulbert and Thompson (10–11 months). In part, this deficiency of existing numerical model is undoubtedly the result of inaccuracies induced in the simulated dynamics by the imposed boundary conditions, which unavoidably, tend to introduce in the system an exaggeratedly strong yearly signal. The discrepancy between observations and simulations, however, may be used to gain a deeper understanding of the subtle dynamics governing the process of ring shedding in the GoM. In fact, it results that as the strength of the yearly signal imposed in realistic model simulations decreases, the occurrence of yearly, ring-shedding events also diminishes. This behaviour may indicate that, in numerical models, a synchronization mechanism exists, which is able to shift a natural ring-shedding period toward a yearly one.

Considering that the seasonal cycle of sea surface temperature in the GoM is a natural forcing on the wide spectrum of physical processes taking place there then emerges an attractive possibility: stochastic resonance. If the imposed forcing by the surface temperature is strong enough to drive the system, we can expect that ring-shedding variability will contain spectral energy in the yearly frequency, but this is not the case, or at least not most of the time. Now, if we include noise as a forcing mechanism, then we can expect that a weak signal in the forcing can be amplified and optimized by the assistance of noise [2]. In other words: if in the GoM exists a weak, yearly signal in the forcing, which is not capable of inducing a yearly period in the ring-shedding process,



**Fig. 2.** Domain used in our model simulations. There are two eastern open boundaries, which are forced by a flow induced by a meridional gradient of upper layer height. These input and output flows are time invariant. Upper panel: height anomalies at day 50. Lower panel: height anomalies at day 500.

this yearly forcing plus a noisy term could be able to produce ring-shedding events with a yearly period.

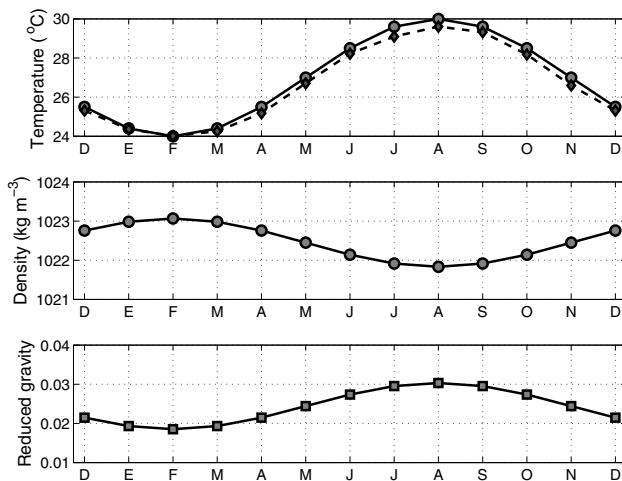
Our conjecture may be tested using simple numerical models. To this purpose, we implemented in the GoM a nonlinear, reduced-gravity model simulating an idealized LC through the inflow/outflow of near-surface water through its boundaries (see Fig. 2). This is the simplest model capable of simulating ring shedding [4]. In this study, it is proposed that a reduced gravity model, where the buoyancy term is seasonally forced and the dissipation term is varied, can explain the observed period of shedding behavior. Additionally, we explore the possibility of stochastic resonance in the ring-shedding process by including a noise term in the seasonally forced buoyancy term.

## 2 Model Simulations

A reduced gravity model (1.5 layers) is used to simulate the ring-shedding process in the GoM and to study the influence of the seasonal cycle upon it. The upper-ocean temperature is assumed constant in space but varies in time. A single sine function is enough to simulate the seasonal variation of the average surface temperature over the GoM (Fig. 3, upper panel). By using this seasonal variation of temperature (while keeping salinity constant) in a linear thermodynamic equation for the density of the upper layer (to make sure that we are being consistent with the planetary geostrophic approximation) we get the seasonal variation of density (Fig. 3, middle panel), which then is used to obtain a reduced gravity parameter that evolves with the time (Fig. 3, lower panel).

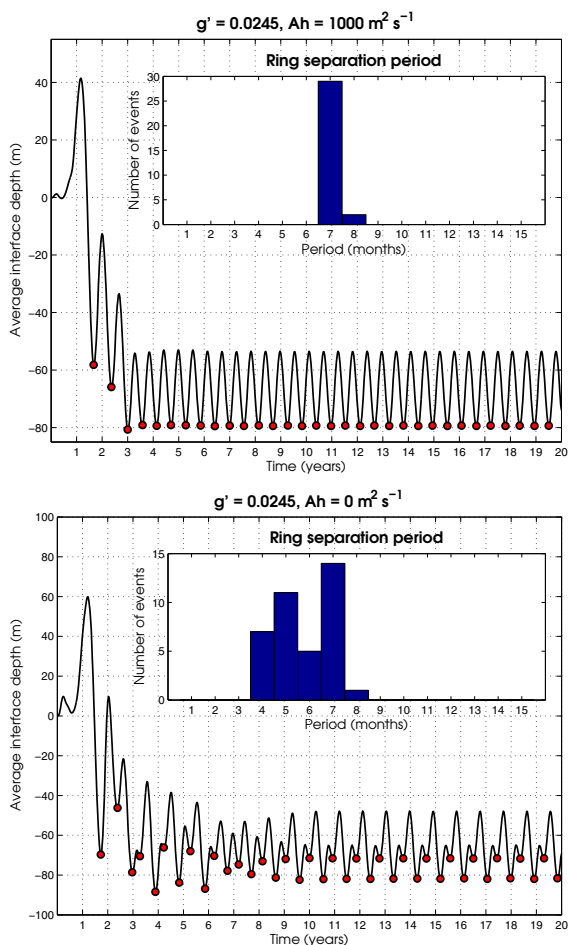
The domain covers the GOM, the Caribbean Sea, and a portion of the Gulf Stream area in the North Atlantic (see Fig. 2). This choice allows the propagation of anomalies from the western Caribbean Sea through the Yucatan Channel and their possible effect on the ring-shedding process. On the other hand, the eastern open boundary is located sufficiently far from our region of primary interest, thus assuring that the adjustment taking place near the open boundary, and the unavoidable reflection of some waves, does not interfere significantly with the internal dynamics of the GoM.

In each model experiment, the equations are integrated numerically for a period of 20 years using a forward Euler scheme and an Arakawa C grid for the active layer. All model experiments are performed using a time step of 90 seconds and a horizontal grid with  $135 \times 153$  points and a spacing of  $1/4$  degree (see model domain in Fig. 2). In a



**Fig. 3.** Upper panel: Average surface temperature over the Gulf of Mexico (95W-84W, 21N-26N) obtained from NOAA Optimum Interpolation Sea Surface Temperature Analysis (dashed line) and seasonal variation of surface temperature simulated by a single sine function (solid line). Middle panel: Density as a linear function of seasonally varying temperature. Lower panel: Seasonal variation of reduced gravity parameter.

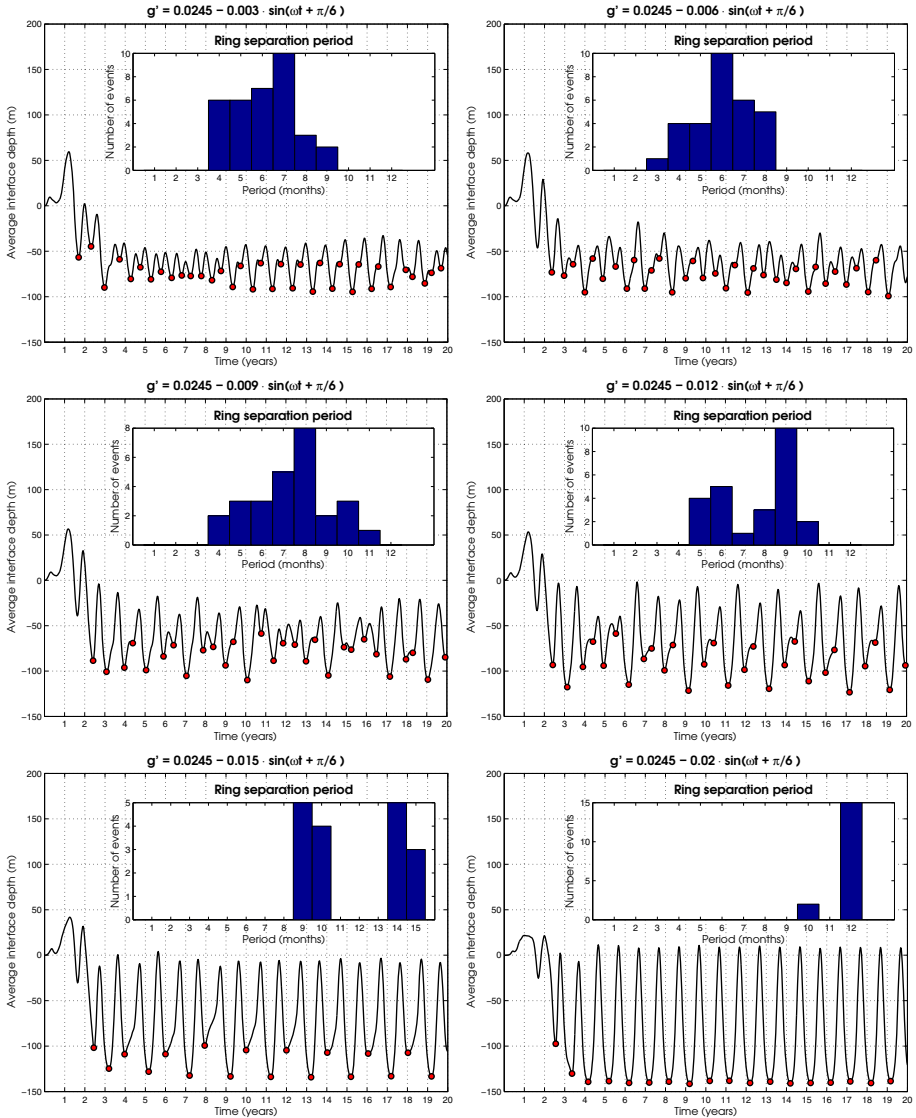
first set of experiments, a constant reduced gravity parameter ( $g'$ ) is used with a value of 0.0245, corresponding to the time-averaged value shown in the lower panel of Fig. 3. In the second set, a seasonally evolving  $g'(t)$  parameter is considered (lower panel of Fig. 3). Finally, in a third set of experiments, a noisy term is added to the time evolving  $g'(t)$  parameter.



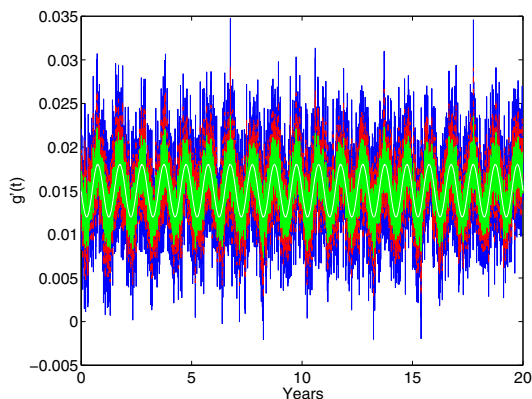
**Fig. 4.** Histograms and time series of area-averaged height anomalies over the ring-shedding region used to estimate the timing of ring shedding events (red dots). By using a diffusion coefficient of  $10^3 \text{ m}^2 \text{ s}^{-1}$ , a constant period of seven months is obtained (upper panel), while with zero diffusion period doubling is observed (lower panel).

### 3 Results

Without seasonal forcing, a single peak in the ring shedding period is observed, which depends on model geometry, upper layer thickness, diffusion, and  $g'$  value (Fig. 4). A red dot in this figure and the following ones indicate the timing of the detachment of a ring of the LC.



**Fig. 5.** Ring-shedding distributions obtained when a seasonal variation in  $g'(t)$  is included. The phase constant is used to obtain a seasonal variation of density in agreement with the seasonal cycle of surface temperature over the central Gulf of Mexico. The amplitude of  $g'(t)$  increases from top to bottom, and from left to right. In all experiments, zero diffusion is used.



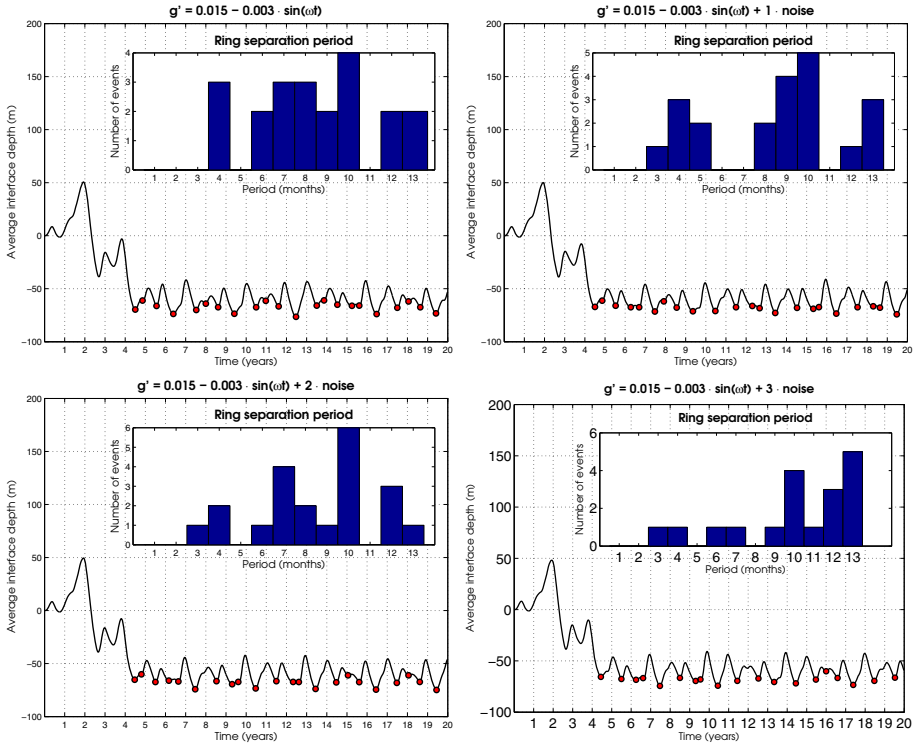
**Fig. 6.** Stochastic forcing. The white curve represents the seasonal variation of  $g'(t)$ . In the green curve, a noise term with zero mean and standard deviation of 0.015 is added. This forcing is multiplied by a factor 2 and 3 in the red and blue curves, respectively.

When the amplitude of the variation of  $g'$  is increased, a more complex behaviour is observed, characterized by a bi-modal distribution, which crucially depends on the amplitude (Fig. 5). When the amplitude is large enough, the annual signal clearly dominates, but when it is weaker different bi-modal distributions appear, some of which resemble the observed one. In the case of large amplitude of the annual signal, a quite simple ring shedding behaviour emerges: constant, yearly ring-shedding events.

Now, we try to answer what happens when the annual signal is weak but high frequency forcing is present. This kind of forcing could be, for example, similar to that associated with turbulent heat fluxes between atmosphere and ocean. A simple way to simulate them is by using a noise term, which represents heat exchange between the ocean surface and the lower atmosphere. In this case, we used for the stochastic forcing a series of random numbers with zero mean and standard deviation of 0.015. Additionally, we explore the effect of noise magnitude on the ring-shedding process using a factor of two and three in the noise term (Fig. 6).

Fig. 7 shows the ring-shedding period distribution resulting of the seasonal variation of  $g'(t)$ . The amplitude of the seasonal forcing is not able to produce yearly ring shedding, because it is not large enough (right-upper panel). The inclusion of a small noise term modifies the distribution of ring-shedding periods but it is not able to induce ring-shedding events with a yearly period.

If the noise amplitude is increased three times, then the noise term is able to change this behaviour, inducing a dominance of ring-shedding events with a period close to the yearly one. It is pertinent to emphasize that looking for the right combinations of  $g'$  values, diffusion, and noise magnitude, it can be achieved a clear dominance of the annual signal.



**Fig. 7.** Ring-shedding period distributions using seasonal forcing (left-upper panel) and seasonal forcing plus noise (right-upper and lower panels). See the text for details.

## 4 Conclusions

In the case of constant  $g'$ , it is shown that large values of diffusion lead to ring-shedding events with a constant period of several months, while with zero diffusion the period is doubled.

By considering a seasonal variation of  $g'(t)$ , a more realistic ring-shedding distribution is obtained, which is characterized by a bi-modal distribution that crucially depends on the amplitude of the seasonal variation.

If the annual signal of  $g'(t)$  has large amplitude, a quite simple ring shedding behaviour is observed: a constant, yearly ring-shedding event.

If the amplitude of  $g'(t)$  is not large enough to induce yearly variability in the ring-shedding process, such behaviour can be obtained by including stochastic forcing with a proper intensity. Thus, it has been shown that stochastic resonance could play a role in the ring-shedding process taking place in the Gulf of Mexico.



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